THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS

Specimen of Written Test at Interview

Issued May 2001

This paper contains questions taken from recent tests and shows the general format. It should not be assumed that the range of topics covered will be the same in December 2001, in particular in light of the changes in A-levels, but the format (and mark allocations) will be similar.

Time allowed: $2\frac{1}{2}$ **hours**

For candidates applying for Mathematics, Computer Science, Mathematics and Computer Science, Mathematics and Philosophy, or Mathematics and Statistics

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Write your name, college (where you are sitting the test), and proposed course (from the list above) in BLOCK CAPITALS.

NAME:

COLLEGE:

COURSE:

Attempt all the questions. Each of the ten parts of Question 1 is worth 4 marks, and Questions 2,3,4 and 5 are worth 15 marks each, giving a total of 100.

Question 1 is a multiple choice question for which marks will be given solely for the correct answer. Answer Question 1 on the grid provided.

THE USE OF CALCULATORS OR FORMULA SHEETS IS PROHIBITED

1. For each part of the question on Pages 3 and 4, you will be given four possible answers just one of which is correct. Indicate for each part A—J which answer (a), (b), (c), or (d) you think is correct with a tick (✓) in the corresponding column in the table below.

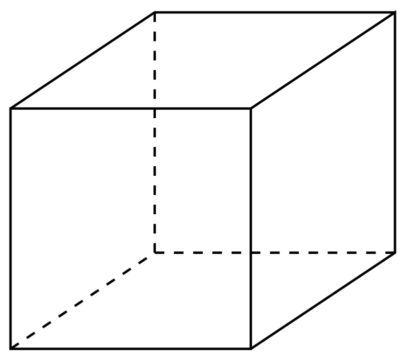
	(a)	(b)	(c)	(d)
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				

A. The substitution $x = y + t$ transforms the equation $x^3 + ax^2 + bx + c = 0$ into an equation of the form $y^3 + py + q = 0$ when

(a) $t = \dfrac{a}{3}$ (b) $t = -\dfrac{a}{3}$ (c) $t = a$ (d) $t = -a$.

B. The faces of a cube are coloured red or blue. Exactly three are red and three are blue. The number of distinguishable cubes that can be produced (allowing the cube to be turned around) is

(a) 2 (b) 4 (c) 6 (d) 20.



C. The shortest distance from the origin to the line $3x + 4y = 25$ is

(a) 3 (b) 4 (c) 5 (d) 6.

D. The numbers 10, 11 and -12 are solutions of the cubic equation

(a) $x^3 - 11x^2 - 122x + 1320 = 0$
(b) $x^3 - 9x^2 + 122x - 1320 = 0$
(c) $x^3 - 9x^2 - 142x + 1320 = 0$
(d) $x^3 + 9x^2 - 58x - 1320 = 0$.

E. The maximum gradient of the curve $y = x^4 - 4x^3 + 4x^2 + 2$ in the range $0 \le x \le 2\frac{1}{5}$ occur when

(a) $x = 0$ (b) $x = 1 - \dfrac{1}{\sqrt{3}}$ (c) $x = 1 + \dfrac{1}{\sqrt{3}}$ (d) $x = 2\dfrac{1}{5}$.

F. The expression $x^2y + xy^2 + y^2z + yz^2 + z^2x + zx^2 - x^3 - y^3 - z^3 - 2xyz$ factorises as

(a) $(x + y + z)(x - y + z)(-x + y - z)$
(b) $(x + y - z)(x - y - z)(-x + y + z)$

(c) $(x+y-z)(x-y+z)(-x+y+z)$
(d) $(x-y-z)(-x-y+z)(-x+y-z)$.

G. The derivative of $xe^{-x^2}\cos\left(\frac{1}{x}\right)$ is

(a) $-\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$
(b) $\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$
(c) $\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) + 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$
(d) $\frac{1}{x}e^{-x^2}\cos\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$.

H. You are *told* that the infinite series $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$ and $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots$ have sums $\frac{\pi^2}{6}$ and $\frac{\pi^2}{8}$ respectively. The infinite series $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots+(-1)^{n-1}\frac{1}{n^2}+\ldots$ has sum equal to

(a) $\frac{\pi^2}{9}$ (b) $\frac{\pi^2}{10}$ (c) $\frac{\pi^2}{12}$ (d) $\frac{\pi^2}{16}$.

I. A grid of size 3cm x 5cm is drawn, ruled at 1cm intervals. The number of squares that can be drawn using the grid is

(a) 15 (b) 18 (c) 26 (d) 37.

J. A pack of cards consists of 52 different cards. A malicious dealer changes one of the cards for a second copy of another card in the pack and he then deals the cards to four players, giving thirteen to each. The probability that one player has two identical cards is

(a) $\frac{3}{13}$ (b) $\frac{12}{51}$ (c) $\frac{1}{4}$ (d) $\frac{13}{51}$

2.

(a) Factorize the expression $x^2 + 3x - 10$.

(b) If $x^3 + ax^2 + bx + c = (x - \alpha)(x - \beta)(x - \gamma)$ for all values of x, find a, b, c in terms of α, β, γ.

(c) Find a value of b for which $x^3 + bx + 2 = 0$ has exactly two distinct solutions.

3.

(a) Find the coordinates of the points at which the two curves $y = 6x^2$ and $y = x^4 - 16$ intersect.

(b) Give a *rough* sketch of the two curves (in the same diagram) for the range $-3 \leq x \leq 3$.

(c) Find the area of the region enclosed by the two curves.

4.

(a) Show that the line $y = mx + c$ passes through the point $(1, 1)$ if $c = 1 - m$.

(b) Let L be a line with gradient $m > 0$, which passes through $(1, 1)$. Find the equation of the line L' which is perpendicular to L, and which passes through the point $(1, a)$, given $a \neq 1$.

(c) Find the area of the triangle which has $(1, 1)$ and $(1, a)$ as two of its vertices and the intersection of L and L' as the third vertex.

(d) For what value of m is the triangle isosceles (two sides of equal length)?

5. A set of 12 rods, each 1 metre long, is arranged so that the rods form the edges of a cube. Two corners, A and B, are picked with AB the diagonal of a face of the cube.

An ant starts at A and walks along the rods from one corner to the next, never changing direction while on any rod. The ant's goal is to reach corner B. A *path* is any route taken by the ant in travelling from A to B.

(a) What is the length of the shortest path, and how many such shortest paths are there?

(b) What are the possible lengths of paths, starting at A and finishing at B, for which the ant does not visit any vertex more than once (including A and B)?

(c) How many different possible paths of greatest length are there in (b)?

(d) Can the ant travel from A to B by passing through every vertex exactly *twice* before arriving at B without revisting A? Give brief reasons for your answer.